PART II: Advanced Physical Limnology

Bertram Boehrer

II-6 circulation of lakes

summer term 2009 — lecture: Wed, 09:15-11:00, INF 229, SR 108
11th Ruprecht-Karls-University Heidelberg, Faculty of Physics and Astronomy

Boehrer and Schultze 2009, Encyclopedia of Inland Waters
Under pressure, water parcels require a smaller volume due to the (small) compressibility of water. The density increase due to compression does not play a role in stability considerations. In-situ density is calculated as:

$$\rho_{\text{situ in}} = \rho (1 - \frac{\rho}{K})$$

Where K is given as a function of T, S, p

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

In the deep water of freshwater lakes, the in-situ density increase due to compression is the leading term, if not otherwise a high gradient is present.

Chen and Millero calculate in-situ density from (potential) density, by adding a pressure term:

$$\rho_{\text{situ in}} = \rho (1 - \frac{\rho}{K})$$

where for freshwater applications

$$\rho_{\text{situ in}} = \rho \left(1 - \frac{\rho}{K}\right)$$

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

A fluid parcel of volume V=\Delta x \Delta t experiences an acceleration defined by the pressure gradient divided by its mass:

$$\frac{\partial}{\partial t} \left( \rho \frac{\partial z}{\partial p} \right) = -\frac{\Delta z}{\Delta t} \frac{\partial \rho}{\partial p}$$

The fluid parcel changes its volume, if there is a horizontal velocity gradient:

$$\frac{\partial}{\partial z} \left( \rho \frac{\partial z}{\partial p} \right) + \frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right)$$

We define (adiabatic) compressibility as

$$\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$$

Hence we find

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$

After cross-differentiation

$$\frac{\partial}{\partial p} \left( \rho \frac{\partial z}{\partial \rho} \right) = \frac{1}{\kappa} \frac{\partial \rho}{\partial p}$$
Transmission of sound

Depth is measured as the travel time of sound and the known sound speed.

Transmission of sound

Lago Llanquihue / Chile

Transmission of sound

Acoustic current measurements

Dushaw et al. 2000

Transmission of sound

Acoustic tomography

http://www.oal.whoi.edu/tomo.html

transmission of light

effects:
- heat input
- energy supply for photosynthesis

Model assumption usually:
\[ I(z) = I_0 e^{-kz} \]
i.e. assumption: no vertical gradients of water properties, no spectral dependence of attenuation.

transmission of light

Vertical gradients and spectral dependence of attenuation.
transmission of light

Transmission

Transmissivity

Turbidity

(potential) density – in-situ density

At temperature of maximum density, the dependence on temperature (at constant pressure and constant salinity) vanishes:

\[
\frac{\partial \rho_s}{\partial T_{\text{max}}} = 0
\]

\[
T_{\text{max}} = 3.9839 - 1.9911 \times 10^{-4} p - 5.822 \times 10^{-10} p^2 - 0.2219 + 1.106 \times 10^{-4} p S
\]

Effect of \( T_{\text{md}} \)

circulation patterns of freshwater lakes

Stability – buoyancy frequency

Moved from its position of neutral buoyancy in a vertical density stratification, a water parcel experiences a force:

\[
F = g \left( \frac{\partial \rho_s}{\partial z} - \frac{\partial \rho}{\partial z} \right)
\]

Which forces an acceleration of

\[
\frac{\partial^2 \rho_s}{\partial z^2} = g \left( \frac{\partial \rho_s}{\partial z} - \frac{\partial \rho}{\partial z} \right)
\]

Without friction, this results in an oscillation at a frequency

\[
\omega = \sqrt{\frac{g}{\rho_s} \left( \frac{\partial \rho_s}{\partial z} - \frac{\partial \rho}{\partial z} \right)}
\]

Assuming the parcel has the same water quality as the surrounding waters, and the only change happening to its density is through the pressure change.

Stability – buoyancy frequency

As a consequence, a potential density is defined with the property:

\[
\frac{\partial \rho_s}{\partial z} = \frac{\partial \rho}{\partial z}
\]

And hence a stability frequency ("Brunt-Väisälä" frequency) [squared], or stability

\[
N^2 = \frac{g}{\rho_s} \left( \frac{\partial \rho_s}{\partial z} - \frac{\partial \rho}{\partial z} \right)
\]

With the assumption from above, that

\[
\rho_{s_{\text{conv}}} = \rho_{s_{\text{conv}}} (T, S, p) \quad \rho_{s_{\text{conv}}} = \rho_{s_{\text{conv}}} (T, S, p)
\]

\[
N^2 = \frac{g}{\rho_s} \left( \frac{\partial \rho_s}{\partial z} - \frac{\partial \rho}{\partial z} \right) \left( N^2 + N^2_{\text{sal}} \right)
\]
Stability – buoyancy frequency

Example Lake Constance:

\[ N^2 = \frac{g \rho}{\rho_0} \frac{\partial \rho}{\partial z} \]

Boehrer 2000, JGR-Oceans


Literature